# Proton Motions in Battery Lead Dioxides 

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#### Abstract

Proton motions in $\alpha$ - and $\beta-\mathrm{PbO}_{2}$ varieties have been studied using neutron quasielastic scattering in the temperature range $77-300 \mathrm{~K}$. Four varieties of $\mathrm{PbO}_{2}$ have been characterized: $\alpha$ and $\beta$ electrochemical $\mathrm{PbO}_{2}$ and two samples obtained from a battery after two types of cycling. Chemically prepared samples were used to compare quasielastic scattering signals. In electrochemical $\mathrm{PbO}_{2}$, two types of motions of protons are clearly evidenced: (i) a rotation-like local motion ( $\mathrm{FWHM}=280 \mu \mathrm{eV}, R=1.1$ $\AA$ ) which disappears at low temperature or after heating and (ii) a translational motion ( $D_{\mathrm{t}} \sim 3 \times 10^{-6}$ $\mathrm{cm}^{2} / \mathrm{sec}$ or, using a jump model, $\tau_{\mathrm{J}}=5 \times 10^{-12} \mathrm{sec}, d_{\mathrm{J}} \sim 1.5$ A). In all samples, a systematic contribution of host lattice is observed in the elastic signals: it might be linked with disordered clusters due to the method of preparation and the presence of protons: two types of elastic contribution can be defined, one connected with some correlations of static distortions. of 1988 Academic Press. Inc.


## I. Introduction

Chemically prepared $\alpha$ - and $\beta-\mathrm{PbO}_{2}$ are reputed to be electrochemically inactive forms. Active forms have to be prepared by electrochemical process directly on the positive plate of lead batteries.

The reasons for the influence of preparation conditions on the electrochemical properties are not well known at present. In addition, recent results seem to indicate that chemical samples should be electrically active ( 1,2 ).

The exact origin of the battery failure, i.e., the loss of capacity due to cycling, is also unknown. In previous works (3), we have established that mechanical factors and chemical and structural changes can play an important role in battery lifetime, the first factors depending probably on the second ones.

An inactive form of $\mathrm{PbO}_{2}$ could be formed during cycling (4); the hypothesis of an electrochemically active amorphous $\mathrm{PbO}_{2}$ form was emphasized. It could be recrystallized during cycling and transformed
into an inactive variety similar to the chemical one.

The origin of the electrochemical activity was also linked with the presence of $H$ species (5). NMR studies have shown the existence of at least two configurations for protons in electrochemically active $\mathrm{PbO}_{2}$ and only one configuration for inactive form and chemically prepared $\mathrm{PbO}_{2}$ (5). NMR as well as inelastic (6) and neutron quasielastic scattering (NQES) studies (3) show that protons are not incorporated in the structure as pure or loosely bound water in pockets.

For $\beta-\mathrm{PbO}_{2}$, the chemical analysis of $\mathrm{Pb}^{4+}$ content by iodometric titration and of total $\mathrm{Pb}\left(\mathrm{Pb}^{4+}+\mathrm{Pb}^{2+}\right)$ by thermogravimetry shows that there is a correlation between H content, measured by neutron transmission, and the difference between the total Pb and $\mathrm{Pb}^{4+}$ contents (7).

Furthermore, a neutron diffraction study (6) has shown that there is no evidence for Pb or O vacancies. In addition, such defects are not necessary for explaining the well-known electrical conductivity of $\mathrm{PbO}_{2}$. As a consequence, it is generally admitted that the chemical formula is $(3,6)$

$$
\begin{aligned}
& \mathrm{Pb}_{1-x / 2}^{4+} \mathrm{Pb}_{x / 2}^{2+} \mathrm{O}_{2-x}^{2-}\left(\mathrm{OH}^{-}\right)_{x} \\
& \text { or } \mathrm{Pb}_{1-x / 2}^{4+} \mathrm{Pb}_{x / 2}^{2+} \mathrm{O}_{2}^{2-} \mathrm{H}_{x}^{+} .
\end{aligned}
$$

Both of these formulas can be associated with two extreme chemical states for hydrogen, or equivalently with two types of motions (local, $\mathrm{OH}^{-}$; jumps, $\mathrm{H}^{+}$). Our previous studies (3) using NQES techniques suggested the existence of at least two types of motions, one of them being a rota-tion-like one.

## II. Structural Background

The $\alpha$ and $\beta$ phases, respectively orthorhombic and tetragonal, have been extensively described (see ( $1,3,6$ )) ; both of
these phases are characterized by a proton content depending on the chemical or electrochemical preparations. However, protons were never localized. The $\alpha$ and $\beta$ phases can be differentiated principally from octahedra packings: the orthorhombic and tetragonal lattices of $\alpha$ and $\beta$ phases are due to different edge sharing of $\mathrm{PbO}_{6}$ octahedra. As a consequence, the shortest $\mathrm{Pb}-\mathrm{O}$ and $\mathrm{Pb}-\mathrm{Pb}$ distances along the two types of $\mathrm{PbO}_{6}$ chains, in $\alpha$ or $\beta$ phases, are not aligned in the same directions in each $\alpha$ or $\beta$ cell. These features are important to understand any structural perturbation due to structural defects. The $\alpha$ phase crystallizes in the orthorhombic system with space group $P b c m$ and $Z=4$ chemical units.

The $\beta$ phase crystallizes in the tetragonal system ( $P 4_{2} / \mathrm{mnm}$ with $Z=2$ : rutile structure). Cell parameters and crystallographic data are given in Table I. The $\beta$ structure is shown in Fig. 1. It consists of columns of edge-sharing $\mathrm{PbO}_{6}$ octahedra. These columns are parallel to the $c$ axis and form sequences of corner-sharing octahedra along the $\langle 110\rangle$ directions. The two apex $\mathrm{Pb}-\mathrm{O}$ distances are slightly shorter than the four equatorial ones. The $a$ and $c$ cell parameters have been found to depend on preparation and cycling. The $a$ parameter is greater for fresh battery picked-up samples


Fig. 1. Structure of the tetragonal $\mathrm{PbO}_{2} \beta$ phase.

TABLE I
Crystallographic Data of the $\beta-\mathrm{PbO}_{2}$ Phase

|  | Sample and <br> techniques | $n(\mathrm{~Pb})$ | $a(\AA)$ | $c(\AA)$ | $x(\mathrm{O})$ | $d\left(\mathrm{O}_{3}-\mathrm{O}_{4}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ref. | $\mathrm{X} ; \mathrm{C}$ |  | 4.965 | 3.387 |  |  |
| $(3)$ | $\mathrm{X} ; \mathrm{E}$ |  | 4.951 | 3.381 |  |  |
| $(3)$ | $\mathrm{N} ; \mathrm{C}$ | $0.986(4)$ | $4.9554(1)$ | $3.3861(1)$ | $0.3068(1)$ | $2.708(1)$ |
| $(6)$ | $\mathrm{N} ; \mathrm{F}$ | $0.954(8)$ | $4.9621(2)$ | $3.3856(2)$ | $0.3052(3)$ | $2.734(3)$ |
| $(6)$ | $\mathrm{N} ; \mathrm{C}$ | $0.986(10)$ | $4.9556(1)$ | $3.3867(1)$ | $0.3066(2)$ |  |
| $(1)$ | $\mathrm{N} ; \mathrm{F}$ | $0.950(16)$ | $4.9642(4)$ | $3.3867(3)$ | $0.3054(5)$ |  |
| $(I)$ | $\mathrm{N} ; \mathrm{F}^{*}$ | $0.967(10)$ | $4.9637(2)$ | $3.3868(1)$ | $0.3063(3)$ | $2.7233(12)$ |
| $(8)$ | $\mathrm{N} ; \mathrm{F}^{* *}$ | $0.978(10)$ | $4.9658(2)$ | $3.3892(1)$ | $0.3060(3)$ | $2.7243(12)$ |
| $(8)$ |  |  |  |  |  |  |

Note. ${ }^{*}$ Before heating; ${ }^{* *}$ after heating at $260^{\circ} \mathrm{C}$, then cooling $\left(20^{\circ} \mathrm{C}\right) . P 4_{2} / \mathrm{mnm}$, $Z=2$; X, X-ray diffraction; N, neutron diffraction; C , chemical sample; E , electrochemical sample; $\mathbf{F}$, fresh battery sample.
than for chemically prepared samples ( 1,6 ). Santoro et al. (6) have shown that the $a$ parameter decreases when the capacity decreases; it should be larger in active forms than in inactive or chemically prepared $\beta-\mathrm{PbO}_{2}$. The $c$ parameter is found to be unchanged.

Another feature is the following: the departure of H species (as $\mathrm{H}_{2} \mathrm{O}$ molecules), by heating at $260^{\circ} \mathrm{C}$ at normal pressure, causes a systematic increasing of both $a$ and $c$ parameters with a strong variation for the latter. This result was first obtained from X-ray diffraction on chemical and electrochemical samples (3) and recently confirmed by neutron diffraction on a fresh battery picked-up sample (8). The principal conclusions of this last study can be summarized as follows:
-Before heating, proton species cannot be located on special sites: a Fourier map was attempted at the end of structural refinements without success.
-After heating at $260^{\circ} \mathrm{C}$ and cooling, the departure of protons is connected with the increase of both $a$ and $c$ parameters (Table I): the $\mathrm{Pb}-\mathrm{O}$ distances remain unchanged while some of the $\mathrm{O}-\mathrm{O}$ distances are modified.
—Only a small Pb deficiency might be defined before and after heating from the site occupation factors; however, the De-bye-Waller factors are large ( $B=1.3$ to 1.6 $\AA^{2}$ ) and the diffraction profiles present small distortions.
-After heating, a continuous and constant decrease of the background is observed all along the diffraction range.

Such results can be understood only if charged proton species are present in the lattice: for example, $\mathrm{H}^{+}$located between two oxygen of an octahedron $\left[\mathrm{PbO}_{6}\right]$ should induce a contraction of $\mathrm{O}-\mathrm{O}$ distance. On the contrary, the presence of neutral water structurally intercalated would have induced an isotropic volume expansion; as a consequence the departure of such neutral molecules after heating would have been associated with a decrease of cell volume, which is not the case.

Considering these results and by analogy with $\mathrm{SnO}_{2}$ and $\mathrm{TiO}_{2}$, we are able to postulate the existence of $\mathrm{OH}^{-}$species substituting for $\mathrm{O}^{2-}$. As we reduce $\mathrm{Pb}^{4+}$ into $\mathrm{Pb}^{2+}$ we suppose that two $\mathrm{OH}^{-}$are located near a $\mathrm{Pb}^{2+}$ ion; such $\mathrm{OII}^{-}$species should be lying along the $\langle 110\rangle$ direction on the common edge of two octahedra (3). As a result
some $\mathrm{O}-\mathrm{H}$. . . O bonds in the $\langle 110\rangle$ direction should be formed and randomly distributed in the structure. Obviously, the other edges of $\mathrm{PbO}_{6}$ octahedra might be visited by the proton which can move more or less rapidly around one oxygen.

The elimination of protons as $\mathrm{H}_{2} \mathrm{O}$ molecules ( $8,10,12$ ) by heating corresponds to the reaction

$$
\begin{aligned}
& \mathrm{Pb}_{1-x / 2}^{4+} \mathrm{Pb}_{x / 2}^{2+} \mathrm{O}_{2-x}^{2-}\left(\mathrm{OH}^{-}\right)_{x} \rightarrow \\
& \quad \mathrm{~Pb}_{\mathrm{I}-x}^{++} \mathrm{Pb}_{x}^{2+} \mathrm{O}_{2-x / 2}^{2-} \square_{x / 2}+\frac{x}{2} \xrightarrow[\mathrm{H}_{2} \mathrm{O}]{ }-x e^{-} .
\end{aligned}
$$

Thus, it causes the formation of some vacancies on oxygen sites and consequently increases the $\mathrm{Pb}^{2+}$ content. This result has two outcomes:
-The $\mathrm{Pb}^{2+}-\mathrm{O}$ bonds are longer (2.21$2.48 \AA$ ) than the $\mathrm{Pb}^{4+}-\mathrm{O}$ bonds (2.13-2.17 $\AA$ ); therefore, the elimination of protons induces a global increase of the cell volume (3).
-The vacancies on oxygen sites induce direct contact between $\mathrm{Ph}^{2+}$ ions in a direction parallel to the $c$ axis ( $3.38 \AA$ ). This extra repulsion makes the cell distortion more important in the $c$ direction than in the $a$ and $b$ directions after heating (3).

Considering all the structural evidence we have postulated the existence of several motions:
(1) a jump between the two oxygens (for example $\mathrm{O}_{3}-\mathrm{O}_{4}$ ) of a common edge; if we considered $d_{\mathrm{OH}} \sim 1.1 \AA$ the jump distance should be $\sim 0.5 \AA$;
(2) a rotation-like motion, on a sphere of radius $R_{0}(\sim 0.9-1.2 \AA)$, around an oxygen atom;
(3) a rotation-like motion on a sphere of radius $R_{\mathrm{Pb}}(\sim 1.5-1.7 \AA)$ if H is supposed to rotate around a Pb atom, all the octahedron edges being visited.

## III. Experiments

## III.1. Origin and Analysis of Samples

Six samples have been investigated and characterized by iodometric titration to determine the $\mathrm{Pb}^{4+}$ content and the quantities of $H$ species present in the structure ( H contents have been confirmed by neutron studies and thermogravimetric analysis $(8,10)$ ):
-Chemical $\alpha-\mathrm{PbO}_{2}$ (referred to as $C_{\alpha}$ ) is prepared by oxidizing lead acetate in solution with an excess of ammonium persulfate in the presence of ammonium acetate and ammonia (7). Its composition corresponds to $x=0.14$.
-Chemical $\beta-\mathrm{PbO}_{2}\left(C_{\beta}\right)$ is a commercial product, "Fluka for analysis," and corresponds to $x=0.05$.
-Electrochemical $\alpha-\mathrm{PbO}_{2}\left(E_{\alpha}\right)$ is prepared by the method described by Nguyen Cong et al. (9); $x=0.10$.
-Electrochemical $\beta-\mathrm{PbO}_{2}\left(E_{\beta}\right)$ is prepared by electrolysis of a 0.5 M solution of $\mathrm{Pb}\left(\mathrm{NO}_{3}\right)_{2}$ in $\mathrm{HNO}_{3}$ and is characterized by $x=0.14$.
-Two samples are picked up from the positive plate of lead batteries after 50 cy cles $\left(B_{50}\right)$ and 860 cycles ( $B_{860}$ ). $B_{50}$ corresponds to the maximal capacity of the battery and $B_{860}$ to its failure ${ }^{1}$ (10). The inhomogeneity (sulfate residue) makes the analysis imprecise for these samples.

## III.2. Quasielastic Neutron Scattering

QENS experiments have been performed on the IN5 spectrometer of the I.L.L. All experiments on electrochemical samples have been performed using the same experimental procedure (beam incidence, Al container, thermal treatment . . .). All samples have been heated previously at $150^{\circ} \mathrm{C}$ for 24 hr to eliminate adsorbed water.

[^0]Samples were stored and transported in the same container under vacuum.

A cryostat has been used in order to observe the evolution of the QES signal when freezing proton motions in the range 77300 K .

Two incident wavelengths were chosen ( $\lambda=5$ and $10 \AA$ ); they allow us to separate two possible types of motions because of two resolutions: $\Delta E \sim 18 \mu \mathrm{eV}$ for $\lambda=10 \AA$ and $\Delta E \sim 120 \mu \mathrm{eV}$ for $\lambda=5 \AA$. In addition for IN5 the elastic scattering profile is triangular so that small Lorentian signals due to QES are easily detectable.

## IV. Results and Discussions

## IV.I. Contribution of Host Lattice

An important feature is observed: the host lattice is responsible for a supplementary signal which is attributed to structural disorder, probably static distortions due to chemical preparation and presence of protons.

This component has been first evaluated by studying two samples ( $C_{\alpha}$ and $C_{\beta}$ ), the compositions of which are well known and quasielastic scattering experiments previously described (3).

The host lattice component has been obtained by extrapolating at $x=0$ the various intensity functions $I(Q)$ for several $Q$ values. The ESFIT5 program permits the separation of the elastic ( $I_{\mathrm{E}}$ ) and quasielastic ( $I_{\mathrm{QE}}$ ) components of signals $I(Q)$. The elastic fraction (EISF) is expressed by

$$
\begin{aligned}
\rho=\frac{I_{\mathrm{E}}}{I_{\mathrm{QE}}+I_{\mathrm{E}}}=\frac{I_{\mathrm{HL}}+I(H)}{I_{\mathrm{QE}}+I_{\mathrm{HL}}+I(H)} & \\
& =b+\frac{I(H)}{I_{\text {Total }}}
\end{aligned}
$$

where $b=I_{\mathrm{HL}} / I_{\text {total }}$ and $I_{\mathrm{HL}}$ is the host lattice elastic scattering (including fixed protons).

If the elastic incoherent structure factor due to mobile protons is $A(Q)=I(H) /\left(I_{\mathrm{QE}}+\right.$
$I(H))$ and $b(Q)$ is the elastic contribution of the disordered host lattice, one can write:

$$
\begin{equation*}
\rho=b+(1-b) \cdot A^{\mathrm{obs}}(Q) \tag{1}
\end{equation*}
$$

For $C_{\alpha}$ and $C_{\beta}$ it has been possible to define a quasiconstant $b$ value in the whole range $0.9<Q<1.9 \AA^{-1}$ out of Bragg peaks,

$$
\begin{equation*}
b=155 /(155+6.74 x) \tag{2}
\end{equation*}
$$

where $x$ is the concentration of H expressed in percentage. For $x=10, b \sim 0.70$; this means that only $30 \%$ of signal is due to protons.

The two samples have been chemically prepared but we have supposed in this evaluation that $b$ is not too dependent on the type of structure ( $\alpha$ or $\beta$ ) nor on the chemical or electrochemical preparation procedure. Thus, the $b$ value obtained from the expression (2) is only an approximate value and is used as a first suitable value for refinement; it will be refined and confirmed later. We have found $b \sim 0.62$ for $E_{\beta}(x=$ $14 \%)$ and $b \sim 0.70$ for $E_{\alpha}(x=10 \%)$. From relation (1) one obtains $A^{\text {obs }}(Q)$ which is related to the geometry of proton motions. Correlatively if the geometry of the motion is known it is possibie to calculate $A(Q)$ then to refine $b$ which is characteristic of elastic scattering from host lattice.

## IV.2. Results at $\lambda=5 \AA$

The jump modet on two sites (13), with a jump distance $d_{\mathrm{J}} \sim 0.5-0.6 \AA$, could not fit the data. A second model has been found to be representative of experimental data. It is the rotational diffusion on the surface of a sphere of radius $R$ (14).

This model depends on a parameter $D_{\mathrm{R}}$. The effective mean HWHM is known to be close $2 D_{\mathrm{R}}$; the FWHM being $4 D_{\mathrm{R}}$ as a first approach.

The EISF is given by

$$
A(Q)=\left(\frac{\sin Q R}{Q R}\right)^{2}
$$

Such a model has allowed us to fit the signals obtained for $\lambda=5 \AA$; it was fitted first for the $E_{\beta}$ sample.

In order to calculate the radius of the rotation, the elastic contribution $b(Q)$ had to be refined. A first calculation was possible using the Debye-Waller (DW) factors obtained from intensities at two temperatures: 150 and 310 K . The following expression is used:

$$
I=I_{0}^{-Q^{2}\langle u\rangle^{2}}=I_{\mathrm{HL}}^{0} e^{-Q^{2}\left\langle u_{\mathrm{HL}}^{2}\right\rangle}+I_{\mathrm{H}}^{0} e^{-Q^{2}\langle\langle\mathrm{H}\rangle},
$$

where $I_{0}=I_{\mathrm{HL}}^{0}+I_{\mathrm{H}}^{0}, b-I_{\mathrm{HL}}^{0} / I_{0}$ and where $\left\langle u^{2}\right\rangle$ is the mean square amplitude of vibration (MSAV). One has the relation used as an approximation,

$$
\begin{aligned}
\left\langle\Delta u^{2}\right\rangle^{\mathrm{obs}} & =\left\langle u^{2}\right\rangle_{310}-\left\langle u^{2}\right\rangle_{150} \\
& =\Delta u_{\mathrm{HL}}^{2} b+(1-b) \Delta u_{\mathrm{H}}^{2}
\end{aligned}
$$

where $\Delta u_{\mathrm{HI} \text {. and }}^{2} \Delta u_{\mathrm{H}}^{2}$ are the variation of MSAV of host lattice ( Pb and O ) and protons.

Observed values of $\left\langle\Delta u^{2}\right\rangle$ for $E_{\beta}$ are given in Table II as a function of $Q$. For Bragg peaks the variation of intensities are principally due to host lattice. Using the

TABLE II
Evolution of $\left\langle\Delta u^{2}\right\rangle$ in the Range 310-150 K as a Function of $Q\left(\AA^{-1}\right)$
and Corresponding Refined $b$
Values for $E_{\beta}$ Sample

|  | $\left\langle\Delta u^{2}\right\rangle$ <br> $(310-150 \mathrm{~K})$ | $b(Q)$ |
| :--- | :---: | :---: |
| $Q$ | 0.47 | $\sim 0.8$ |
| 0.34 | 0.15 | 0.002 |
| 0.53 | 0.076 | 0.36 |
| 0.74 | 0.076 | 0.43 |
| 0.94 | 0.070 | 0.61 |
| 1.134 | 0.071 | 0.67 |
| 1.319 | 0.050 | 0.76 |
| 1.494 | 0.049 | 0.78 |
| 1.66 | 0.017 | 0.987 |
| 1.81 | 0.026 | 0.857 |
| 1.93 | 0.034 | 0.823 |
| 2.1 |  |  |

Bragg peaks we obtained $\Delta u_{\mathrm{HL}}^{2}=0.010 \AA^{2}$ (for $Q=1.81 \AA^{-1}$ ).

The observed $\left\langle\Delta u^{2}\right\rangle$ values allow us to find a mean $\Delta u_{\mathrm{H}}^{2}$ value and to evaluate the effective $b$ values for each $Q$. Using thesc refined $b$ values it is then possible to define a unique $\left\langle\Delta u_{\mathrm{H}}^{2}\right\rangle$ value: $\left\langle\Delta u_{\mathrm{H}}^{2}\right\rangle=0.15 \AA^{2}$.

The $b(Q)$ effective value being determined, the fit is operated in fixing $R$ at various values lying in the range $0.8-2 \AA$ and refining the $D_{\mathrm{R}}$ parameter from the rotational diffusion model of Sears. The model is considered to be well fitted when $D_{\mathrm{R}}$ values, calculated for each $Q$ values, are quasiconstant; this criterion was satisfied for $R=1.1 \pm 0.1 \AA$. Under these conditions the mean $D_{\mathrm{R}}$ value is found equal to $70 \mu \mathrm{eV}$ which corresponds to $\mathrm{FWHM}=280 \mu \mathrm{eV}$.

Figure 2 represents some of the fits. Table III gives the results of last calculations obtained for the $E_{\beta}$ sample. The elastic fraction obtained from a model independent fit is recalled; the observed and calculated val-


FIg. 2. Characteristic fits for $\lambda=5 \AA . D_{\mathrm{R}}$ is constant and $b$ is refined.

TABLE III
Quasielastic Data at $\lambda=5 \AA$

| $Q\left(\AA^{-1}\right)$ | $\rho_{\text {fi }}$ | $A_{\text {obs }}$ | $A_{\text {calc }}$ |
| :--- | :--- | :--- | :--- |
| 0.53 | 0.970 | 0.970 | 0.89 |
| 0.74 | 0.874 | 0.800 | 0.80 |
| 0.94 | 0.820 | 0.680 | 0.69 |
| 1.134 | 0.838 | 0.584 | 0.58 |
| 1.319 | 0.823 | 0.660 | 0.47 |
| 1.494 | 0.834 | 0.300 | 0.368 |
| 1.66 | 0.846 | 0.309 | 0.280 |
| 1.81 | $0.990^{a}$ | 0.230 | 0.210 |
| 1.93 | 0.885 | 0.196 | 0.156 |
| 2.10 | 0.850 | 0.152 | 0.102 |

Note. Fits for $E_{\beta}$ sample at $30^{\circ} \mathrm{C}\left(D_{\mathrm{R}}=70 \pm\right.$ $\left.20 \mu \mathrm{eV}, R=1.1 \pm 0.2 \AA, A_{\text {calc }}=(\sin Q R / Q R)^{2}\right)$.
${ }^{a}$ Bragg peak.
ues $A^{\text {obs }}$ and $A^{\text {calc }}$ are compared. The program then allows us to refine the individual $b(Q)$ values for $R$ and $D_{\mathrm{R}}$ fixed. These values are found to be close to the initial ones.

In Table IV we have gathered the refined elastic scattering component $b(Q)$ for the four samples $E_{\beta}, E_{\alpha}, B_{50}$, and $B_{860}$ at 310 K and for $E_{\beta}$ at 150 K . Even measurements including Bragg peaks have been treated. We may note that $b$ for $Q>1 \AA^{-1}$ (out of Bragg peaks) is greater for $E_{\alpha}$ than for $E_{\beta}$ and for $B_{860}$ than for $B_{50}$. This means that the contribution duc to protons is, relative to the host lattice one, less important for $E_{\alpha}$, which has less proton ( $10 \%$ ), than for $E_{\beta}(14 \%)$ and less important for $B_{860}$ (end of life) than for $B_{50}$ (fresh battery).
IV.3. Results at $\lambda=10 \AA$

We have taken into account the previous results obtained at $\lambda-5 \AA$. At $\lambda=10 \AA$ the elastic contribution of the total signal is such that:

$$
\begin{aligned}
b^{*}(Q) & =\frac{I_{\mathrm{HL}}^{\prime}}{I_{\text {Total }}(\lambda=10 \AA)} \\
& =\frac{I_{\mathrm{HL}}-I_{\mathrm{H}}^{(2)}}{I_{\mathrm{HL}}+I_{\mathrm{H}}^{(2)}+I_{\mathrm{H}}^{(1)}}=\frac{b}{\rho(5 \AA)}-\frac{I_{\mathrm{H}}^{(2)}}{\rho(5 \AA)} .
\end{aligned}
$$

In this expression $I_{\mathrm{H}}^{(2)}$ is the term describing the second proton motion, while $I_{\mathrm{H}}^{(1)}$ is that obtained at $\lambda=5 \AA$ (rotation); at $\lambda=$ $10 \AA$ the $I_{Q E}$ component due to rotation has vanished in the background. The effective host lattice contribution is $I_{\mathrm{HL}}^{\prime}$. As a first approach the term $I_{\mathrm{H}}^{(2)} / \rho$ was neglected.

Using the $b(Q)$ values at $\lambda=5 \AA$ it has been possible to evaluate the $b^{*}$ elastic term (for $\lambda=10 \AA$ ) in the range $0-1 \AA^{-1}$; under that condition a small quasielastic contribution has been detected at 310 K . This contribution disappears at 150 K .

In the range $Q=0.15-0.4 \AA^{-1}$ the elastic contribution is strong (Figs. 3 and 4). However, between $Q=0.5$ and $Q=1.2 \AA^{-1} \mathrm{a}$ weak increasing quasielastic signal is observed in each sample. Table V gathers the principal results obtained at $\lambda=10 \AA$. The $I(Q)$ values are represented in Figs. 3 and 4. Figure 5 shows the observed HWHM for each sample. At $Q=0.7 \AA^{-1}$, HWHM ~ $10 \pm 2 \mu \mathrm{eV}$, and at $Q=1.2 \AA^{-1}$, HWHM ~ $25 \pm 5 \mu \mathrm{eV}$.

The systematic evolution should be as-

TABLE IV
Refined Elastic Components $b$ at $\lambda=5 \AA$ for Four Samples as a Function of $Q$

|  | $E_{\beta}$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\left.\AA^{-1}\right)$ |  | 310 K | 150 K |  | $E_{r y}$ |
|  |  | $B_{50}$ | $B_{860}$ |  |  |  |
| 0.32 | 0.72 | 0.93 | 0.85 | 0.82 | 0.92 |  |
| 0.36 | 0.09 | 0.001 | 0.30 | 0.42 | 0.57 |  |
| 0.53 | 0.002 | 0.005 | 0.00 | 0.19 | 0.006 |  |
| 0.74 | 0.36 | 0.86 | 0.705 | 0.55 | 0.79 |  |
| 0.94 | 0.43 | 1.00 | 0.93 | 0.85 | 0.92 |  |
| 1.134 | 0.61 | 0.51 | 0.71 | 0.69 | 0.80 |  |
| 1.319 | 0.67 | 0.54 | 0.72 | 0.72 | 0.81 |  |
| 1.494 | 0.76 | 0.59 | 0.74 | 0.75 | 0.82 |  |
| 1.66 | 0.78 | 0.71 | 0.92 | 0.82 | 0.85 |  |
| 1.81 | 0.99 | 1.00 | 0.87 | 0.98 | 0.98 |  |
| 1.93 | 0.86 | 0.93 | 0.87 | 0.82 | 0.87 |  |
| 2.10 | 0.82 | 0.16 | 0.87 | 0.83 | 0.89 |  |
| $\langle b\rangle^{h}$ | $0.68(8)$ | $0.54(4)$ | $0.72(2)$ | $0.72(3)$ | $0.81(1)$ |  |

[^1]

Fig. 3. Intensities observed at $\lambda=10 \AA$ for $E_{\alpha}$ and $E_{\beta}$ samples as a function of $Q^{2}$.
cribed to some translational motion because of the apparent $Q^{2}$ variation of HWHM. Two types of motions can be postulated.
(a) a continuous translation through the lattice with HWHM $=D_{t} Q^{2}$ so that

$$
D_{\mathrm{t}} \sim(3 \pm 1) \times 10^{-6} \mathrm{~cm}^{2} \mathrm{sec}^{-1}
$$

(b) a jump model (the Chudley-Elliot model) with jump distances $d_{\mathrm{J}}$ corresponding to cubic configuration and with residence time $\tau_{\mathrm{J}}$.

TABLE V
Quasielastic Scattering at $\lambda=10$ $\AA$ (Fit of One Lorentzian): Elastic Contribution of Host Lattice (b) and HWHM in $\mu$ ev

| $Q\left(\AA^{-1}\right)$ | $\left\langle b^{a}(Q)\right\rangle$ | HWHM $(\mu \mathrm{eV})$ |
| :---: | :---: | :---: |
| 0.15 | 0.95 | $<1$. |
| 0.27 | 0.70 | 1. |
| 0.48 | 0.20 | $2 . \pm 1$. |
| 0.68 | 0.75 | $6 . \pm 3$. |
| 0.95 | 0.95 | $20 . \pm 10$. |
| 1.07 | 0.85 | $20 . \pm 15$. |

${ }^{a} b$ values are the average values for the four samples.


Fig. 4. Intensities observed at $\lambda=10 \AA$ for $B_{50}$ and $B_{860}$ samples as a function of $Q^{2}$.

$$
S(Q, \omega)=\frac{1}{\pi} \frac{f(Q)}{f^{2}(Q)+\omega^{2}}
$$

$$
\mathrm{FWHM}=2 f(Q)=\frac{1}{\tau_{\mathrm{J}}}\left[1-\frac{\sin Q d_{\mathrm{J}}}{Q d_{\mathrm{J}}}\right] .
$$

Considering the same motion for each sample one obtains:

$$
\begin{aligned}
\tau_{\mathrm{J}} & =(0.5 \pm 0.3) \times 10^{-11} \mathrm{sec} \\
d_{\mathrm{J}} & =1.5 \pm 0.5 \AA .
\end{aligned}
$$



FIg. 5. HWHM as a function of $Q$ for $\lambda=10 \AA$.

In Fig. 5 a small tendency for larger diffusion rate is observed in the case of cycled samples but due to the incertainty it is not possible to draw conclusions about it. Using the $Z_{\mathrm{J}}$ and $d_{\mathrm{J}}$ values it is possible to calculate a $D_{\mathrm{t}}$ coefficient (cubic configuration):

$$
D_{\mathrm{t}}=\frac{d_{\mathrm{J}}^{2}}{6 \tau_{\mathrm{J}}}=7.2 \times 10^{-6} \mathrm{~cm}^{2} \mathrm{sec}^{-1}
$$

With a $d_{1}$ limit value of $1 \AA$ one obtains $D_{1} \sim$ $3.2 \times 10^{-6} \mathrm{~cm}^{2} \mathrm{sec}^{-1}$ which is in the range of $D_{\mathrm{t}}$ values for continuous translational motions.

The mean $d_{\mathrm{J}}$ value can be compared with expected jump distances on octahedron edges. Along the $c$ axis the jump $\left(\mathrm{O}_{3}-\mathrm{O}_{5}\right)$ can be evaluated using

$$
c=2 d_{\mathrm{OH}}+d_{\mathrm{J}}=3.38 \AA ;
$$

with $d_{\mathrm{OH}}=1.1 \AA$ (rotation radius) we obtain $d_{\mathrm{J}} \sim 1.18 \AA$.

From one edge to another, a jump distance in the range $0.9-1.5 \AA$ can be expected. As a consequence successive jumps all along octahedron edges and along the $c$ direction can be expected; this can be described by mean $\left\langle d_{\mathrm{J}}\right\rangle$ values of about I to $1.5 \AA$.

## IV.4. Proton Population and Host Lattice Contribution

The quasielastic signals at $\lambda=5$ and $10 \AA$ have been treated independently. At $5 \AA$, in the range $1<Q<2 \AA^{-1}$ out of Bragg peaks, the $b$ value is slowly increasing: at $Q=1.3$ $\AA^{-1}, b \sim 0.65$. At $\lambda=10 \AA$ the $b^{*}$ value is calculated after postulating $b^{*}=b / p$; for $Q=0.74 \AA^{-1}, b^{*} \sim 0.5$, and for $Q=1.1$ $\AA^{-1}, b^{*} \sim 0.73$.

The $b$ and $b^{*}$ values can be compared to the following expressions:

$$
\begin{aligned}
b(Q) & \simeq \frac{K \sigma_{\mathrm{PbO}_{2}}\left(1-D_{\mathrm{S}}\right)+\sigma_{\mathrm{H}}^{\text {inc. }} x P_{2} \mathrm{DW}}{K \sigma_{\mathrm{PbO}_{2}}\left(1-D_{\mathrm{S}}\right)+\sigma_{\mathrm{H}}^{\text {inc } \cdot} x P_{2} \mathrm{DW}+\sigma_{\mathrm{H}}^{\text {inc }} x P_{P_{1}} \mathrm{DW}} \\
b^{*}(Q) & =\frac{K \sigma_{\mathrm{PbO}_{2}}\left(1-D_{\mathrm{S}}\right)+\sigma_{\mathrm{H}}^{\text {inc. }} x P_{2} \mathrm{DW}}{K \sigma_{\mathrm{PbO}_{2}}\left(1-D_{\mathrm{S}}\right)+\sigma_{\mathrm{H}}^{\text {incc }} x P_{2} \mathrm{DW}+\sigma_{\mathrm{H}}^{\text {inc}} x P_{1} \mathrm{DWA}(Q)},
\end{aligned}
$$

where $\sigma_{H}^{\text {inc. }}=80, x \sim 0.14$ (atomic concentration in the well known $\beta$ phase $E_{\beta}$ ), $K$ is the proportion of host lattice involved in distortions, $\sigma_{\mathrm{PbO}}=19.3$ is the total (incoherent + coherent) scattering contribution of one formula $\mathrm{PbO}_{2}, D_{\mathrm{S}}$ is the static distortion term considered as the approximate function $D_{\mathrm{S}}=e^{-Q^{2}\left\langle u_{\mathrm{i}}^{2}\right\rangle},\left\langle u_{\mathrm{st}}^{2}\right\rangle$ represents the disordered distortions due to defects, DW is the Debye-Waller term associated with proton vibrations ( $e^{-Q^{2}\left\{u_{i \dagger}^{2}\right\rangle}$, where $\left\langle u_{\mathrm{th}}^{2}\right\rangle$ is taken to be equal to $0.30 \AA^{2}$ ), $P_{1}$ and $P_{2}$ are the populations of protons involved in independent motions (rotations and translations, respectively).

In these expressions, correlated motions
have not been considered because of poor statistics and large uncertainties concerning all models; thus, $P_{1}+P_{2}=1$.

The principal difficulty was to evaluate the contribution of elastic diffuse scattering (EDS); apparently the first undulation of the $b(Q)$ function suggests the existence of some correlations in distortions. However, we tried to evaluate the $D_{\mathrm{S}}$ term by fitting a crude function to the experimental points $b(Q)$ at $Q \sim 0.5$ and $1.3 \AA^{-1}$; we have obtained $u_{\mathrm{st}}^{2} \sim 0.5 \pm 0.3 \AA^{2}$.

Then using the values of $b$ and $b^{*}$ respectively at $Q=1.3 \AA^{-1}$ and at $Q=0.74$ or 1.1 $\AA^{-1}$ it was possible to obtain the order of magnitude for $P_{1}, P_{2}$, and $K$ :

$$
\begin{aligned}
& K=0.5 \pm 0.3, \quad P_{1}=0.65 \pm 0.2 \\
& P_{2}=0.35 \pm 0.2
\end{aligned}
$$

In other terms the host lattice might deliver an incoherent signal of one-half of the total scattering; among protons in motions two-thirds should be engaged in rotations ( $P_{1}$ ) and about one-third in continuous translations $\left(P_{2}\right)$.

## IV.5. Small Angle Scattering and Proton Concentration

Small angle scattering has been observed for all the samples. An example is given in Figs. 3 and 4 in the case of $\lambda=10 \AA$ for the four studied samples. First, in Fig. 3 we note that the intensity is more important for $E_{\beta}$ for which proton content is greater ( $x=$ 0.14 instead of $x=0.10$ for $E_{\alpha}$ ). A difference is also noted between $B_{50}$ and $B_{860}$. A direct comparison is difficult between the two sets of samples (battery samples and electrochemical samples) but it is probable that $B_{860}$ contains less protons than $B_{50}$. Such a hypothesis has already been emphasized.

These observations have to be correlated with the relative contribution of the host lattice to the incoherent signal: $b(\beta)<b(\alpha)$ for $x(\beta)>x(\alpha)$ and $b\left(B_{50}\right)<b\left(B_{860}\right)$.

The intensity at small angle is probably related to the presence of clusters as suggested previously (3). The density of clusters seems directly connected with the proton concentration of the samples.

In the picked-up samples, $B_{50}$ and $B_{860}$, the evolution of signals with cycling, argue in favor of cluster density decrease, i.e., proton elimination during cycling.

Figure 6a represents the general variations observed for $b(Q)$; a systematic minimum is observed at $Q=0.5 \AA^{-1}$ for each sample. An undulation of $b(Q)$ might be defined at $Q>1 \AA^{-1}$. In Figs. 6b and 6 c , the small angle scattering and the EDS term due to coherent scattering from distortions are represented (rough estimates). The su-


FIg. 6. Elastic contribution of host lattice $(\lambda-5 \AA)$ as a function of $Q$.
perposition of both features might explain our present observations. It is important to note that the maximum at $Q-0.95 \AA^{-1}$ has been confirmed in experiments at $\lambda=10 \AA$ (Figs. 3 and 4).

## V. Conclusion

## V.I. Types of Motions

Table VI summarizes the principal results obtained in this study. We have established the existence of at least two types of proton motions in agreement with the two chemical formulas mentioned under the Introduction:
(1) a local motion similar to an $\mathrm{OH}^{-}$ hindered rotation (radius, $1.1 \AA$ );
(2) a long-range translation motion which can be either continuous ( $D_{\mathrm{t}} \sim 3 \times$ $10^{-6} \mathrm{~cm}^{2} \mathrm{sec}^{-1}$ ) or not (residence time, $\tau_{\mathrm{J}}=$ $0.5 \times 10^{11} \mathrm{sec}$, jump distance $\sim 1.5 \AA$ ).

If two types of protons are postulated the population of protons in rotation is about $65 \%$ while that of protons in long-range translation should be about $35 \%$.

TABLE VI
Summary or Principal Results at $\lambda=5 \AA$ and $\lambda=10 \AA$ for the Rotation Model ( $R$ ), the Continuous Translation Model ( $D_{4}$ ), and the Chudley-Elliot Model ( $d_{\mathrm{J}}, \tau_{\mathrm{J}}, D_{\mathrm{T}}$ )

| Characteristics | $\lambda=5 \AA$ | $\lambda=10 \AA$ |
| :--- | :--- | :--- |
| FWHM $(\mu \mathrm{eV})$ | 280. | $Q$ dependent |
| $R(\AA)$ | $1.1 \pm .2$ |  |
| $D_{1}\left(\mathrm{~cm}^{2} \sec { }^{1}\right)$ |  | $3 . \pm 1.10^{-6}$ |
| $d_{\mathrm{J}}(\AA)$ |  | $1.5 \pm 0.5$ |
| $\tau_{\mathrm{J}}(\mathrm{sec})$ |  | $0.510^{-11}$ |
| $D_{\mathrm{T}}\left(\mathrm{cm}^{2} \mathrm{sec}^{-1}\right)$ |  | $7.210^{-6}$ |
| $P_{i}(\%)$ | $65 \pm 20$ | $35 \pm 20$ |
| $K(\%)$ | $50 \pm 30$ | $50 \pm 30$ |

Note. $P_{i}$ and $K$ are, respectively, the contribution of each movement and of the host lattice (see text). The given data are the average value obtained for all the samples.

The translational motion with $D_{1}=3 \times$ $10^{-6} \mathrm{~cm}^{2} \mathrm{sec}^{-1}$ can be directly compared with the value recently found by Münzberg and Pohl (II) for the chemical diffusion coefficient of hydrogen in $\mathrm{PbO}_{2}$ obtained from the electrochemical method at various pHs and concentrations of $\mathrm{Pb}^{2+}$ :

$$
\tilde{D}_{\mathrm{H}} \sim 0.4 \text { to } 4.9 \times 10^{-7} \mathrm{~cm}^{2} \mathrm{sec}^{-1}
$$

Such a coefficient is linked to the $D_{\mathrm{t}}$ coefficient by the relation

$$
\tilde{D}_{\mathrm{H}}=D_{\mathrm{t}} \frac{d \ln a}{d \ln c}
$$

where $a$ and $c$ are, respectively, the activity of protons and the concentration of $\mathrm{Pb}^{2+}$ within the solid, i.e., of proton in the oxide. The order of magnitude of the experimental $D_{\mathrm{t}}$ coefficient can be considered to be in agreement with $\tilde{D}_{\mathrm{H}}$, which is necessarily smaller than $D_{\mathrm{t}}$.

In addition, it has heen observed as an important contribution of the host lattice to the elastic signals: at small $Q$ values, strong elastic components indicate the presence of small clusters. Such clusters are probably domains with smaller specific volume and
in which protons are agglomerated. The long-range motion ( $D_{\mathrm{t}}$ ) might be connected with migration between clusters. Inside cluster zones, protons might be trapped with rotation-like motions.

In electrochemical samples protons should be more dilute than in chemical samples (3). The cycling procedure of batteries should be associated with elimination of "free protons" (the long-range diffusion signal was not observed in chemically prepared samples (3)); this cycling should induce condensation in larger domains or elimination from the bulk due to diffusion through grain surfaces.

The host lattice is found to be disordered (large $b$ values $\sim 50 \%$ for the global contribution for both $\lambda=10 \AA$ and $\lambda=5 \AA$ spectra). This disorder is directly connected with the presence of clusters.

## V.2. Clusters and Chemical Reactivity

Let us recall the condensed chemical formula of lead dioxide:

$$
\mathrm{Pb}_{1-x / 2}^{4+} \mathrm{Pb}_{x / 2}^{2+} \mathrm{O}_{2}^{2-} \mathrm{H}_{x}^{+} .
$$

The presence of clusters is evidenced by the following developed formula (if we first consider that all protons are trapped in the clusters):

$$
\left(1-\frac{x}{2}\right)\left[\mathrm{Pb}^{4+} \mathrm{O}_{2}^{2-}\right]+\frac{x}{2}\left[\mathrm{~Pb}^{2+}\left(\mathrm{OH}^{-}\right)_{2}\right]
$$

Two types of domains appear: the lattice of composition $\mathrm{PbO}_{2}$ and the clusters of composition $\mathrm{Pb}(\mathrm{OH})_{2}$. The volume ratio of the two types of domains is $(x / 2) /(1-x / 2)$. When the lead dioxide is heated the clusters of composition $\mathrm{Pb}(\mathrm{OH})_{2}$ probably first decompose into PbO clusters because of the lower thermal stability of $\mathrm{Pb}^{2+}-(\mathrm{OH})^{-}$ bonds:

$$
\mathrm{Pb}^{2+}(\mathrm{OH})_{2}^{-} \rightarrow \mathrm{Pb}^{2+} \mathrm{O}^{2-}+\widetilde{\mathrm{H}_{2} \mathrm{O}}
$$

This process has been confirmed by mass spectroscopy measurements; the first step
of decomposition is characterized by departure of water molecules ( 8,10 ). The influence of protons (trapped in clusters or isolated) on the chemical reactivity, especially on reduction (in monoxide or sulfate), is yet to be postulated $(5,12)$.

So as a first step of decomposition we have the juxtaposition of a matrix of composition $\mathrm{PbO}_{2}$ and microzones of composition PbO . This juxtaposition is possible because the packing of lead atoms is the same for all lead oxides in the range $\mathrm{PbO}_{2}-\mathrm{PbO}$ (except for $\mathrm{Pb}_{2} \mathrm{O}_{3}$ and $\mathrm{Pb}_{3} \mathrm{O}_{4}$ (12)), the electron lone pairs of $\mathrm{Pb}^{2+}$ ions playing the stereochemical role of oxygen atoms.

Thus, the existence of zones of different compositions according to the density of clusters is coherent with our previous model of decomposition (12). The intermediary pseudocubic lead oxides $\mathrm{PbO}_{x}(x=$ 1.57 and $x=1.41$ ) may be interpreted thanks to the presence of different zones: for example, $\mathrm{PbO}_{1.57}$ could be considered to be the juxtaposition of zones of composition $\mathrm{Pb}_{24} \mathrm{O}_{40}$ and $\mathrm{Pb}_{24} \mathrm{O}_{36}$ (12).

We have seen that all protons are not trapped in clusters but are involved in translational motions. Such protons are disseminated in the bulk. Both types of protons probably induce displacements of lead atoms (the $\mathrm{Pb}^{2+}$ ions), which do not scatter coherently with the host lattice. Therefore the effective diffraction pattern should be due to a more complex average structure corresponding to the mean formula

$$
\mathrm{Pb}_{1-x / 2}^{4+} \mathrm{Pb}_{x / 2}^{2-} \mathrm{O}_{2-x}^{2-}\left(\mathrm{OH}^{-}\right)_{x}
$$

where the index $x / 2$ corresponds to virtual vacancies because of the displacements of $\mathrm{Pb}^{2+}$ from the theoretical site $\left(\mathrm{Pb}^{4+}\right)$.

This may explain the apparent Pb vacancies observed in our own neutron diffraction refinements (Table I).

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[^0]:    ${ }^{1}$ Samples given by C.E.Ac.

[^1]:    Note. The starting values are $b_{0}=0.62$ for the $\beta$ variety and $b_{0}=0.695$ for the $\alpha$ one. The $B_{50}$ and $B_{860}$ samples are principally of the $\beta$ variety.
    ${ }^{4}$ Close to $Q=0.94 \AA^{-1}$ an anomaly is observed in the case of $E_{\alpha}, B_{50}, B_{860}$ samples.
    ${ }^{h}$ Averaged $b$ values obtained in the range $1<Q<1.5 \AA \AA^{\prime}$.

